Disability Employment Levy-Grant Scheme from an Economic Viewpoint

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This paper discusses the disability employment quota-levy policy from an economic point of view. Based on the standpoint that the goal of the policy is to promote employment of disabled people in the society as a whole and the employment quota for each firm should be adjusted according to its capacity or opportunity cost associated with the employment, we underline that the levy-grant system can be utilized as an instrument that provides each firm with incentives to spontaneously achieve its optimal employment level. It is demonstrated that levy and grant can work as Pigouvian tax under complete information and that a levy-grant scheme akin to the Vickrey auction mechanism achieves efficiency even when the cost structure of the firms is private information.

I. Introduction

In Japan, there are 7.2 million people with intellectual, mental and physical disabilities (Cabinet Office, Annual Report on Government Measures for Persons with disabilities in 2008). For the 3.9 million disabled people of age 18 or over, the Ministry of Health, Labour and Welfare (MHLW) introduces several employment promotion schemes, such as the *Employment Quota System for Persons with Disabilities* (EQSPD) and the *Levy and Grant System for Employing People with Disabilities* (LGSEPD). Under these systems, employers with 56 workers or more should employ persons with intellectual or physical disabilities at least as many as 1.8% of the total employees; upon failing, those with 301 workers or more have to pay a *levy* of 50,000 yen per person a month for the number short of the quota;[1] otherwise, they receive a *grant* of 27,000 yen per person a month (independent of the size).

One way to view these levies and grants is that

- a law should be obeyed, and
- a levy is a punishment to employers who fail to obey the law, while a grant is a reward to those who follow it.

Differentiating from such a view, we base our argument on the view that

- the goal of the employment policy for disabled persons is to promote a certain level of total employment in the society as a whole, and
- different firms should employ different numbers of disabled persons depending on

[1] After July 2010, employers with 201 workers or more will have to pay a levy as well, but no levy is imposed on employers with 300 workers or less for the moment.

their capacity.

From this point of view, we show that the levy-grant scheme can lead employers to achieve the optimal employment *voluntarily*.

Only 325,000 out of 3.9 million disabled persons aged 18 or over are employed by firms with 56 workers or more (MHLW, Situations of Employments for Persons with Disabilities in 2008).[2] The actual employment rate of disabled persons is 1.59% in 2008, still lower than the socially required rate 1.8%, and only 44.9% of firms achieve the employment rate 1.8%. The current situation thus suggests that we need to further promote employment of disabled persons. In this paper, we discuss optimal disability employment levy-grant schemes from an economic viewpoint.

II. Analysis of the Levy-Grant Scheme

As we noted, the current policy uniformly imposes an employment quota of rate 1.8% on all firms with 301 workers or more. In this section, we demonstrate that such a uniform quota is socially inefficient, resulting in wastes of resources and hence losses in social welfare. The point is that it fails to appreciate the fact that different firms have different capacities in employing disabled people due to their businesses, facilities, or other various factors. For example, consider employing a physically disabled person who uses a wheelchair. Facilities then needed to be installed would be significantly different whether the person is employed as an office worker to work at a desk or he is employed as a factory worker to engage in manufacturing work. The difference in capacities thus leads to the difference in costs to bear upon employment, which in turn entails the difference in optimal employment across different firms. In what follows, we formalize this point based on a simple theoretical model.

1. Simple Case

In order to understand the essence of our analysis, let us first consider the simplest case where there are only two firms (*A* and *B*) of same size and two disabled persons. As a matter of fact, other things being equal firms would prefer to employ a person having no disability rather than a disabled person; indeed, even with the current regulation, the employment rate for the disabled lies below 1.8%. For example, when employing a disabled person a firm may have to install facilities that would not have been necessary if it had employed a person with no disability, and in some businesses, the productivity will inevitably be lowered. In economic terms, these are counted as *opportunity costs* (or simply, costs) of employing disabled persons. Let c_1^A denote the cost for firm *A* of employing one disabled person and c_2^A denote that of employing an additional one; thus, firm *A* incurs cost c_1^A to employ one person and

[2] 448,000 disabled persons are employed by firms with 5 workers or more (MHLW, Survey of Situations of Employments for Persons with Disabilities in 2008).

$c_1^A + c_2^A$ to employ two persons. Similarly, let c_1^B and c_2^B denote the costs for firm *B* of employing a first and a second disabled persons, respectively. These costs, of course, depend on the nature of the business. The opportunity cost may be almost zero (and can even be negative, that is, the benefit can be positive) when the disabled employee conducts office work, while it can be huge when the work is one of construction. From now on, we assume that $c_1^A, c_2^A < c_1^B, c_2^B$.

Let $\bar{x}$ be the employment quota, the number of disabled persons that each firm is required to employ. We assume that $\bar{x} = 1$, so that the target total employment is $2\bar{x} = 2$.

If the firms comply with the employment quota, the social cost is equal to $c_1^A + c_1^B$. In contrast, the socially efficient employment in the sense of total cost minimization is that firm *A*, the firm with lower costs, employs both of the two persons. In this case, the total cost is $c_1^A + c_2^A$, which is lower than $c_1^A + c_1^B$ by the assumption that $c_2^A < c_1^B$.

Now suppose that the authority applies levy T^* and grant T^{**} such that $c_2^A < T^*$, $T^{**} < c_1^B$. Then, firm *A* is willing to employ two persons and receive a grant T^{**}, since

$$c_1^A > c_1^A + c_2^A - T^{**},$$

so that the grant compensates the cost of employing the second person. On the other hand, firm *B* prefers to pay a levy T^* instead of employing one person, since

$$c_1^B > T^*,$$

so that for firm *B* the levy is smaller than the opportunity cost of employment. In this way, social efficiency obtains as a result of voluntary, or even opportunistic, decisions of the firms once levies and grants are properly designed. It has to be noted that this levy-grant scheme achieves the social target, employment of two disabled persons, by socially efficient employment here, as opposed to the previous case of employment quota.

2. The Model

We more generally consider a case where there are *n* firms of same size. Denote as previously by $\bar{x}$ the employment quota for each firm, so that the social target of employment is $n\bar{x}$. Let $c(x^i; \theta^i)$ be the (total) cost for firm *i* of employing x^i persons with disabilities, where the parameter θ^i represents the cost structure of firm *i*. For simplicity, we regard x^i as a continuous variable. We assume that $c_x > 0$, $c_{xx} > 0$, and $c_{x\theta} > 0$. Thus, a lower θ corresponds to a more efficient firm (i.e., a firm with lower marginal costs). Each firm *i* determines x^i so as to minimize its cost.

Here, we assume that the authority completely knows the firms' cost functions. The objective of the authority is to achieve employment of $n\bar{x}$ disabled persons while minimizing the sum of the costs the *n* firms incur. We consider the Pigouvian taxation where the authority charges each firm *i* a levy *t* per person if x^i is below $\bar{x}$ and subsidizes a grant *t* per person if x^i exceeds $\bar{x}$.

3. Analysis

First, let us derive the socially optimal employment. The authority's minimization problem is given by

$$\min_{x^1,\dots,x^n} \sum_{i=1}^{n} c(x^i;\theta^i)$$

$$\text{s.t.} \sum_{i=1}^{n} x^i = n\bar{x}.$$

Assuming the solution lies in the interior, the first order condition is given by

$$c_x(x^{1*};\ \theta^1) = \cdots = c_x(x^{n*};\ \theta^n)$$

along with the constraint $\sum_{i=1}^{n} x^{i*} = n\bar{x}$. Notice that, by the assumption that $c_{x\theta} > 0$, if $\theta^i < \theta^j$, then $x^{i*} > x^{j*}$. That is, at the social optimum, more efficient firms should employ more persons than otherwise.

Now suppose that the authority exercises the Pigouvian taxes as described above. The minimization problem of each firm *i* is then

$$\min_x c\,(x;\ \theta^i) + t(\bar{x} - x).$$

The first order condition is given by

$$c_x(x;\ \theta^i) = t.$$

Thus, by setting the levy-grant level t^* so that

$$t^* = c_x(x^{i*};\theta^i)$$

for all *i*, the authority can achieve the efficient employment ($x^{1*},\dots, x^{n*}$).

Importantly, that the society as a whole achieves employment of disabled people at rate 1.8% does *not* imply that each firm should uniformly achieve employment at rate 1.8%. Uniform employment would force extra costs, giving rise to social inefficiencies. In contrast, levies and grants, appropriately designed, drive firms to voluntarily determine their employment levels through their own optimization problem, as a consequence of which the socially efficient employment is attained. The levy-grant scheme can thus be utilized as a means of achieving social efficiency.

III. The Optimal System under Asymmetric Information

As we discussed above, it is socially inefficient to impose a uniform employment quota on all firms. At the social optimum, firms with lower marginal costs should employ more disabled persons than those with higher costs. A properly designed levy-grant scheme will work to attain the social optimum.

In practice, however, it is not a trivial task for the authority to determine the optimal levels of levy and grant; as opposed to the premise in the previous analysis, the authority most likely does not have complete knowledge about the cost structures of the firms, that is, they are *private information* of the firms. Notice that the result in the previous section crucially depends on the assumption that the firms' cost functions are known to the authority. One might think, then, that the authority should just tell the firms to inform it of their cost functions, for example upon annual reporting of their employment. However, firms will have no incentive to report their cost functions truthfully, unless the authority appropriately designs a mechanism to induce the firms to do so.

1. Gradual Adjustment of Levies and Grants

One way to attain the desirable employment and social efficiency under private information is to adjust the levies and grants gradually by trial and error until the employment reaches the desirable level. The current employment rate of the disabled, 1.59% in 2008, is fairly below 1.8%, suggesting that the authority should raise the amounts of levies and grants.

In fact, they have been raised. The levy has been set 30,000 yen in 1977, 40,000 yen in 1980, and 50,000 yen in 1991; and the grant 14,000 yen in 1977, 20,000 yen in 1980, 25,000 yen in 1991, and 27,000 yen in 2003 (Nakajima, Nakano, and Imada 2005). Despite the increase in these instruments, the employment rate has remained largely unchanged, which suggests that they should be raised further. Eventually, the target employment of rate 1.8% will indeed be attained by adjusting the levies and grants in every period based on the rate in the previous one (notice that even in this case, firms will not establish the same average employment).

However, such a policy of gradual adjustment is hardly an ideal one, since adjustment is likely to take a very long time, and the social situation may change during the time. Moreover, anticipating a future increase in grants, firms may strategically postpone their employment. In order to achieve the target employment immediately, therefore, the authority anyway needs to obtain information about firms' cost structures to establish an appropriate institution. Is it possible, then, for the authority to design such an institution giving firms right incentives to report their private information truthfully? In fact, auction theory (more generally, the theory of *mechanism design*) helps to solve this issue. In the following sections, we study a mechanism called the Vickrey-Clarke-Groves mechanism, especially the Vickrey auction.[3]

[3] Vickrey (1961) proposed the second-price auction for both cases of single and multiple objects. Clarke (1971) studied a similar mechanism for provision of public goods, while Groves (1973) unified and generalized these ideas.

2. Auction

How do auctions relate to our issue of determining the optimal levy and grant levels under private information? Consider selling one unit of an object through an auction. Specifically, we consider the sealed-bid second-price auction, also known as the Vickrey auction.[4] In a Vickrey auction, the bidder with the highest bid wins the auction (i.e., obtains the object) and pays the second highest bid, rather than the bid he himself submitted. A participant is thus unsure ex ante how much he will pay in case he wins. What is the optimal bid for a participant in the Vickrey auction? Let v^i be participant i's valuation of the object, and denote his bid by b^i. The Vickrey auction is known to have a nice property that $b^i = v^i$ is a weakly dominant strategy for each participant i. That is, *it is optimal for every participant to bid his valuation truthfully*. Hence, the Vickrey auction achieves efficiency in that the object is obtained by the participant who values it most. See any textbook on auction theory, e.g., Krishna (2002), for details. The point is that the winner pays the price equal to the valuation of the participant that would have obtained the object if he were absent. Note that the price the winner pays does not depend on the bid he made.

Let us apply the Vickrey mechanism to our problem. Consider the case where there are n firms, 1, …, n, and one disabled person to be employed. Let c^i be the cost for firm i, which is firm i's private information, and suppose that the firms report their costs to the authority. Denote firm i's report by b^i, and rename the firms so that $b^1 \leq b^2 \leq \cdots \leq b^n$. According to the idea of the Vickrey auction, the scheme that serves for our goal is such that firm 1 which makes the smallest report employs the person and then either (i) each of firms 2, …, n pays an amount b^1 of levy, or (ii) firm 1 receives an amount b^2 of grant.[5]

Exactly as in the auction situation above, under this scheme it is optimal for each firm to report its cost truthfully. By applying the Vickrey mechanism, the authority can thus induce the firms to report truthfully and hence achieve social efficiency in that the firm with the lowest cost will employ the person.

In order to deal with the more realistic case where there are more than one disabled persons to be employed, we consider the multi-object Vickrey auction in the next subsection.

3. Vickrey Auction with Multiple Objects

We first explain the multi-object Vickrey auction (see, e.g., Krishna 2002). Suppose that there are K units of a homogeneous object to be sold through an auction. Each participant i submits a profile of his marginal valuations of the first through the Kth units, i.e., a bid vector $\mathbf{b}^i$ of K elements, which can be different from his true valuation profile $\mathbf{v}^i$ (where we

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[4] More familiar is perhaps the first-price auction, where the bidder with the highest bid obtains the object and pays the price he submitted.

[5] More generally, the scheme works such that each firm i pays an amount T_i^* of levy when it is not to employ the person and receives an amount T_i^{**} of grant when it is to employ, where T_i^* and T_i^{**} are such that $T_i^* + T_i^{**} = \max_{j \neq i} b^j$.

assume $v_1^i \geq v_2^i \geq \cdots \geq v_K^i$). In other words, he reports his demand curve, a list of his willingness to pay for the first unit, the second unit, …, and the *K*th unit. Based on these bids, the objects are allocated to the bidder with the highest bid, the one with the second highest, …, and the one with the *K*th highest. Thus, bidder *i* obtains k^i units of the object if k^i elements of his bid vector $\mathbf{b}^i$ are in the top *K* bids among all the bids. The payment of bidder *i* is given by

$$\sum_{k=1}^{k^i} b_{K-k^i+k}^{-i}, \tag{1}$$

where $\mathbf{b}^{-i}$ is the vector of all bids of all bidders except i and its elements are arranged in a descending order. That is, he is charged the amount of the highest rejected bid (other than his own) for his first unit, the second highest rejected bid for his second unit, …, and the k^ith highest rejected bid for his k^ith unit. The basic idea here is the same as that of the Vickrey auction in the single object case: bidder *i* is charged the amount of the bids by the bidders who would have obtained the k^i objects if *i* were absent.

The Vickrey auction is known to have the following nice property.

Theorem 1. In the Vickrey auction, $\mathbf{b}^i = \mathbf{v}^i$ is a weakly dominant strategy for each participant *i*.

The theorem says that no participant can gain a positive benefit by making a false report (i.e., by submitting a list of marginal valuations that are different from his true ones), and as a result, the bids are precisely the participants' true marginal valuations. We therefore have the following.

Corollary 2. The Vickrey auction allocates the objects efficiently.

That is, the objects are allocated to the participants who value them most. The Vickrey auction thus succeeds in maximizing the social surplus, hence achieving social efficiency, even in the presence of private information.

In the next subsection, applying the multi-object Vickrey auction explained above, we propose a mechanism that achieves socially efficient employment of disabled persons.

4. Vickrey Auction Employment Scheme

We consider a case where there are *n* firms of same size. Denote as previously by $\bar{x}$ the employment quota for each firm, so that the social target of employment is $n\bar{x}$. Let $\mathbf{c}^i = (c_1^i, \ldots, c_{n\bar{x}}^i)$ be the cost vector for firm *i*, where an element c_j^i represents the marginal cost of firm *i* to employ the *j*th disabled person, which is firm *i*'s private information, and we assume that $c_1^i \leq c_2^i \leq \cdots \leq c_{n\bar{x}}^i$. That is, for firm *i*, employing one disabled person costs c_1^i, employing two disabled persons costs $c_1^i + c_2^i$, and so on. We consider the following procedure.

(i) Given $\mathbf{c}^i$, each firm *i* reports a vector of marginal costs $\mathbf{b}^i = (b_1^i, \ldots, b_{n\bar{x}}^i)$.

(ii) Arrange all the elements of the reports in an ascending order. Firm *i* employs x^i disabled persons if x^i elements in its report are in the top $n\bar{x}$ elements among all the reports.

(iii) If firm *i* employs $x^i > \bar{x}$ disabled persons, the grant for firm *i* is $\sum_{x=1}^{x^i-\bar{x}} b^{-i}_{n\bar{x}-x^i+x}$, where $\mathbf{b}^{-i}$ is the vector of all reports of all firms except i and its elements are arranged in an ascending order.

(iv) If firm *i* employs $x^i < \bar{x}$ disabled persons, the levy for firm *i* is $\sum_{x=0}^{\bar{x}-x^i-1} b^{-i}_{n\bar{x}-x^i-x}$.

As previously, we obtain the following result.

Proposition 3. In the Vickrey scheme, $\mathbf{b}^i = \mathbf{c}^i$ is a weakly dominant strategy for each firm *i*.

The proposition says that no firm can gain a positive benefit by making a false report $\mathbf{b}^i \neq \mathbf{c}^i$, and thus the firms report their cost structures truthfully. We therefore have the following.

Corollary 4. The Vickrey scheme achieves the efficient employment.

That is, the outcome of the scheme minimizes the total cost of employment.

In order to see how the Vickrey scheme works, let us consider the following simple example.

Numerical example

Consider the case where there are three firms (i.e., $n = 3$). Suppose $\bar{x} = 2$, thus the social target is $n\bar{x} = 6$. The cost vectors for the firms are given by

$$\mathbf{c}^1 = (1, 3, 4, 6, 9, 10)$$
$$\mathbf{c}^2 = (2, 5, 6, 7, 8, 9)$$
$$\mathbf{c}^3 = (1, 4, 7, 10, 11, 12).$$

Let $\mathbf{b}^i$ be the report vector of firm *i*, and $\mathbf{b}$ be the vector of reports of all firms and its elements are arranged in an ascending order. When all firms truthfully report their costs, we obtain

$$\mathbf{b} = (\underline{1}, \underline{1}, \underline{2}, \underline{3}, \underline{4}, \underline{4}, 5, 6, 6, 7, 7, 8, 9, 9, 10, 10, 11, 12),$$

and

$$\mathbf{b}^1 = (\underline{1}, \underline{3}, \underline{4}, 6, 9, 10)$$
$$\mathbf{b}^2 = (\underline{2}, 5, 6, 7, 8, 9)$$
$$\mathbf{b}^3 = (\underline{1}, \underline{4}, 7, 10, 11, 12),$$

where the 6 smallest "bids" are underlined. Thus, in this case, firm 1 employs 3 persons, firm 2 employs 1 person, and firm 3 employs 2 persons. Firm 1 is given a grant for its third employee, which equals 5, the fourth element of

$$\mathbf{b}^{-1} = (1, 2, 4, \underline{5}, 6, 7, 7, 8, 9, 10, 11, 12).$$

If firm 1 did not employ the third person, then firm 2 would have employed one additional person incurring cost 5, as firm 2's report contains the smallest "rejected bid" in $\mathbf{b}^{-1}$.

On the other hand, a levy of 4 is imposed on firm 2 since

$$\mathbf{b}^{-2} = (1, 1, 3, 4, \underline{4}, 6, 7, 9, 10, 10, 11, 12).$$

If firm 2 employed the second person, then firm 1 (or firm 3) would not have employed the third (or second) person, saving cost 4.

The remaining firm 3 is assigned no levy or grant since it employs $\bar{x}$ (= 2) persons.

The important point is that no firm can manipulate the amount of levy or grant by making a false report, since these are determined by the *other* firms' reports. Consequently, the Vickrey scheme achieves the efficient employment even if the authority does not know the actual cost structure of each firm.

IV. Conclusion

According to the Employment Quota System for Persons with Disabilities, firms have to offer equal employment opportunities to people with disabilities as well as those having no disabilities. The actual employment is still lower than the socially desired rate 1.8%, which suggests that we need to further promote employment of disabled persons. We argued that the current policy of *uniform* quota is inappropriate since it leads to inefficiency, giving rise to wastes of resources and hence losses in social welfare. Different firms have different capacities for employing disabled persons due to the difference in their businesses, facilities, and so on, so that firms with lower employment costs should employ more persons than otherwise in order for the social cost to be minimized. Levies and grants can thus be used as an instrument for Pigouvian taxation, provided that the firms' cost structures are known to the authority.

When the cost structures are firms' private information, one has to design an appropriate mechanism that gives the firms incentives to reveal their private information. We proposed such a mechanism building on the insights from *mechanism design theory*. There, levies and grants are designed based on the idea of the Vickrey auction so that firms have incentives to report truthfully their marginal costs for employing disabled persons, and as a consequence, employees are efficiently assigned to firms with the lowest marginal costs. One shortcoming of this mechanism is that the amount of information to be processed increases rapidly as the numbers of firms and employees increase. It may thus be practical to use this mechanism at a provincial level.

We restricted ourselves to simple cases to isolate the theoretical essence of the optimal levy-grant scheme, and hence we neglected many factors. First, we focused exclusively on pecuniary motives derived by levies and grants. Non-pecuniary factors such as reputation effects may also discipline firms to employ disabled people.[6] Second, we considered only the incentives of employers and abstracted from heterogeneity of employees. People are heterogeneous in their characteristics or preferences and hence desirable jobs. Matching between

[6] However, Nagae (2007) reports evidence from the disclosure policy in Tokyo and Osaka that reputation does not effectively work for promoting employment of the disabled.

employers and employees is without doubt an important issue. Finally, our discussion has been static. Obviously, investment in infrastructures will decrease future costs of employment of disabled people. Such dynamic effects should be taken into account. Further research is called for on these issues.

Appendix: Proof of Theorem 1

Consider any bidder i, and fix the vector $\mathbf{b}^{-i}$ of bids of all bidders except bidder i, arranged in a descending order. Denote by $u^i(\mathbf{b}^i;\ \mathbf{v}^i)$ bidder i's (ex post) payoff when he submits a vector $\mathbf{b}^i$. Let k^i be the number of units that bidder i obtains when he submits $\mathbf{v}^i$; that is, k^i is the number of elements in $\mathbf{v}^i$ that are in the top K bids among all the bids. It therefore holds that $v^i_k \geq b^{-i}_{K-k^i+1}$ for all $k \leq k^i$ and $v^i_k \leq b^{-i}_{K-k^i}$ for all $k > k^i$. Bidder i's payoff upon truth-telling is given by

$$u^i(\mathbf{v}^i;\ \mathbf{v}^i) = \sum_{k=1}^{k^i} v^i_k - \sum_{k=1}^{k^i} b^{-i}_{K-k^i+k}.$$

Now consider any $\mathbf{b}^i \neq \mathbf{v}^i$. We want to show that i does not obtain a positive gain by submitting $\mathbf{b}^i$, i.e., $u^i(\mathbf{b}^i;\ \mathbf{v}^i) \leq u^i(\mathbf{v}^i;\ \mathbf{v}^i)$. Let ℓ^i be the number of units that i receives when he submits $\mathbf{b}^i$. Then the payoff is given by

$$u^i(\mathbf{b}^i;\ \mathbf{v}^i) = \sum_{k=1}^{\ell^i} v^i_k - \sum_{k=1}^{\ell^i} b^{-i}_{K-\ell^i+k}.$$

There are three cases: (i) $\ell^i = k^i$, (ii) $\ell^i > k^i$, and (iii) $\ell^i < k^i$.

Case (i): i is assigned the same number of units and the same amount of payment (recall that the payment does not depend on one's own bid), so that his payoff becomes no larger than that under truth-telling.

Case (ii): The payoff is computed as

$$\begin{aligned} u^i(\mathbf{b}^i;\ \mathbf{v}^i) &= \left(\sum_{k=1}^{k^i} v^i_k + \sum_{k=k^i+1}^{\ell^i} v^i_k\right) - \left(\sum_{k=1}^{k^i} b^{-i}_{K-k^i+k} + \sum_{k=-(\ell^i-k^i)+1}^{0} b^{-i}_{K-k^i+k}\right) \\ &= u^i(\mathbf{v}^i;\ \mathbf{v}^i) + \sum_{k=k^i+1}^{\ell^i} \left(v^i_k - b^{-i}_{K-\ell^i-k^i+k}\right) \\ &\leq u^i(\mathbf{v}^i;\ \mathbf{v}^i) + \sum_{k=k^i+1}^{\ell^i} \left(v^i_k - b^{-i}_{K-k^i}\right) \leq u^i\ (\mathbf{v}^i;\ \mathbf{v}^i), \end{aligned}$$

where the last inequality follows from $v^i_k \leq b^{-i}_{K-k^i}$ for all $k > k^i$. That is, for the $(k^i + 1)$th through the ℓ^ith units, i has to pay more than his valuation.

Case (iii): The payoff is computed as

$$u^i(\mathbf{b}^i;\ \mathbf{v}^i) = \left(\sum_{k=1}^{k^i} v_k^i - \sum_{k=\ell^i+1}^{k^i} v_k^i\right) - \left(\sum_{k=1}^{k^i} b_{K-k^i+k}^{-i} - \sum_{k=1}^{k^i-\ell^i} b_{K-k^i+k}^{-i}\right)$$

$$= u^i(\mathbf{v}^i;\ \mathbf{v}^i) - \sum_{k=\ell^i+1}^{k^i} \left(v_k^i - b_{K-\ell^i-k^i+k}^{-i}\right)$$

$$\leq u^i(\mathbf{v}^i;\ \mathbf{v}^i) - \sum_{k=\ell^i+1}^{k^i} \left(v_k^i - b_{K-k^i+1}^{-i}\right) \leq u^i\ (\mathbf{v}^i;\ \mathbf{v}^i),$$

where the last inequality follows from $v_k^i \geq b_{K-k^i+1}^{-i}$ for all $k \leq k^i$. That is, for the $(\ell^i + 1)$th through the k^ith units, *i* saves less than his valuation.

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